

# Neutrino-pair emission due to electron-phonon scattering in a neutron star crust: a reappraisal

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## Abstract

The process of  $\nu\bar{\nu}$  radiation due to interaction of electrons with phonons in the crust of a cooling neutron star is studied with the consistent account of an electromagnetic coupling between electrons in the medium. The wavelength of radiated neutrinos and antineutrinos is typically much larger than the electron Debye screening distance in the medium, and therefore plasma polarization substantially modifies the effective weak current of the electron. It is shown, that under above conditions plasma polarization screens totally a vector weak interaction of the electron with a neutrino field. As a result, the  $\nu\bar{\nu}$  emissivity is less in approximately 2.23 times than previously estimated.

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Due to neutrino cooling, the inner crust of a newborn neutron star crystallizes during a very short time, while thermal relaxation is achieved in about  $1 \div 10^3$  years. The relaxation process is accompanied by cooling waves propagated from the stellar core to the surface, and dynamics of relaxation is very sensitive to neutrino energy losses in the crust of a neutron star [1]. The melting temperature of the crust is much less than the electron plasma frequency<sup>1</sup>. Therefore, the plasmon decay processes as well as the photoproduction of neutrino pairs are suppressed exponentially, and bremsstrahlung of electrons becomes the basic mechanism for production of neutrino pairs in the crystalline crust. Band structure effects suppress exponentially the  $\nu\bar{\nu}$  bremsstrahlung caused by electron scattering in a static lattice [2]. Therefore, the basic contribution to neutrino-pair radiation is due to interaction of electrons with phonons in the crystal, i.e. to an absorption and creation of phonons by an electron with simultaneous emission of a neutrino pair. This process has been previously studied by Flowers and Itoh et al. [3], [4], [5] by the standard formalism of the electron-phonon interaction [6]. The basic assumption common to these calculations is that the in-medium weak interaction of the electron with a neutrino field may be treated as that for vacuum. By this assumption, they have included into the matrix element of the corresponding reaction only two of Feynman's diagrams (1a and 1b) shown in Fig. 1. In these diagrams, the broken line is a phonon created or absorbed by the electron, and the filled rectangle corresponds to in-vacuum weak coupling of electrons with the neutrino field. What we demonstrate in this paper is that contributions into the matrix element of the next two diagrams (1c and 1d) shown in Fig. 1 is not negligible. In these diagrams, the dashed line is a virtual in-medium photon, and the neutrino-pair is radiated by ambient electrons through polarization of the plasma caused by the quantum transition of the initial electron. At the first sight, the latter diagrams contain an additional small factor  $e^2 = 1/137$ . However, the fine structure constant enters the matrix element as a combination with the electron number density  $n_e$  known as

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<sup>1</sup> In what follows we use the system of units  $\hbar = c = 1$  and the Boltzmann constant  $k_B = 1$ .

the Debye screening distance  $D_e$ . In the considering case of degenerate ultrarelativistic electron gas one has

$$D_e \simeq \sqrt{\frac{\pi}{4e^2}} \frac{1}{p_F},$$

where  $p_F = (3\pi^2 n_e)^{1/3}$  is the electron Fermi momentum. Therefore, the relevant parameter of the problem is  $k^2 D_e^2$ , where  $k$  is the momentum carried out by the neutrino-pair. As will be shown, when  $k^2 D_e^2 \lesssim 1$ , the diagrams 1c and 1d of Fig. 1 are not small and must be necessarily included in the matrix element of reaction. The physical reason of this can be imagined as follows: by absorbing (or creating) a phonon, the initial electron electromagnetically induces some motion of the other electrons inside the Debye sphere around itself. If the wavelength  $\lambda$  of radiated neutrino and antineutrino is larger than the electronic Debye screening distance, then the weak current of perturbed electrons inside the Debye sphere generate neutrinos coherently with the weak current of the initial electron. The energy of radiated neutrino-pairs is of the order of the medium temperature, which is assumed to be less than the melting temperature  $T_m$  of the crystal. The melting temperature of a classical one component crystal of ions of a charge  $Z$  is equal to [7]

$$T_m \simeq \frac{Z^2 e^2}{172} \left( \frac{4\pi n_i}{3} \right)^{1/3},$$

with  $n_i = Z n_e$  being the number density of ions. Thus, one has

$$k^2 D_e^2 \lesssim \frac{T^2}{T_m^2} (T_m^2 D_e^2) \simeq 2.4 \times 10^{-2} \left( \frac{Z}{50} \right)^{\frac{10}{3}} \frac{T^2}{T_m^2},$$

and the condition  $k^2 D_e^2 \ll 1$  is fulfilled. Therefore, the diagrams 1c and 1d of Fig. 1 must be included in the matrix element of the reaction. More precisely, this can be proved in the following way. From the diagrams shown in Fig. 1, it is obvious, that to take into account the indicated collective effects, it is necessary to replace in the previously made calculations [3], [4], [5] the vacuum weak coupling by the effective weak coupling of an electron with a neutrino field in the medium. This effective interaction represents the total of two diagrams shown in Fig. 2.

We use the Standard Model of weak interactions and consider low-energy fermions, typical for neutron star interiors, therefore the weak interaction of an electron with a neutrino field can be written in a point-like current-current approach

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu J^\mu,$$

where  $G_F$  is the Fermi coupling constant. The effective electron weak current is the sum

$$J^\mu = j_{vac}^\mu + j_{ind}^\mu$$

of the weak current of an electron in vacuum (the first diagram of Fig. 2) and the induced weak current of ambient electrons caused by the plasma polarization. The vacuum weak current of the initial electron is of the standard form

$$j_{vac}^\mu = \bar{\psi} \gamma^\mu (c_V - c_A \gamma_5) \psi, \quad (1)$$

where  $\psi$  stands for electron field;  $c_V = \frac{1}{2} + 2 \sin^2 \theta_W$ ,  $c_A = \frac{1}{2}$  for emission of electron neutrinos, and  $c'_V = -\frac{1}{2} + 2 \sin^2 \theta_W$ ,  $c'_A = -\frac{1}{2}$  for muon and tau neutrinos;  $\theta_W$  is the Weinberg angle. The second diagram of Fig. 2 is the contribution of the weak current of ambient electrons caused by the plasma polarization. The electron loop in the second diagram can be expressed using the polarization tensors of the electron gas. Thus,

$$j_{ind}^\mu = \frac{1}{4\pi} \bar{\psi} \left[ \gamma^\lambda D_{\lambda\rho} (c_V \Pi^{\rho\mu} - c_A \Pi_5^{\rho\mu}) \right] \psi,$$

where  $D_{\lambda\rho}(K)$  is the photon propagator in the medium;  $\Pi^{\mu\rho}(K)$  and  $\Pi_5^{\mu\rho}(K)$  are the plasma polarization tensors, which depend on the four-momentum transfer  $K = (\omega, \mathbf{k})$ . An extra factor  $(4\pi)^{-1}$  appears here because the factor  $4\pi$  is traditionally included into the definition of polarization tensors (see lower). The corresponding factor  $e^2$  which appears from diagrams is also included into the definition of the polarization tensors. Thus, the total effective weak current is of the following form

$$J^\mu = \bar{\psi} \left[ \gamma^\mu (c_V - c_A \gamma_5) + \frac{1}{4\pi} \gamma^\lambda D_{\lambda\rho} (c_V \Pi^{\rho\mu} - c_A \Pi_5^{\rho\mu}) \right] \psi. \quad (2)$$

The plasma polarization tensors, defined in the one-loop approximation, can be written as follows

$$\Pi^{\mu\rho} = 4\pi ie^2 \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \gamma^\mu \hat{G}(p) \gamma^\rho \hat{G}(p + K), \quad (3)$$

$$\Pi_5^{\mu\rho} = 4\pi ie^2 \text{Tr} \int \frac{d^4 p}{(2\pi)^4} \gamma^\mu \hat{G}(p) \gamma^\rho \gamma_5 \hat{G}(p + K). \quad (4)$$

Here  $\hat{G}(p)$  is the in-medium electron Green's function. To specify the components of the polarization tensor, we select a basis constructed from the following orthogonal four-vectors

$$h^\mu \equiv \frac{(\omega, \mathbf{k})}{\sqrt{K^2}}, \quad l^\mu \equiv \frac{(k, \omega \mathbf{n})}{\sqrt{K^2}},$$

where the space-like unit vector  $\mathbf{n} = \mathbf{k}/k$ ,  $k = |\mathbf{k}|$  is directed along the electromagnetic wave vector  $\mathbf{k}$ . Thus, the longitudinal basis tensor can be chosen as  $L^{\rho\mu} \equiv -l^\rho l^\mu$ . The transverse (with respect to  $\mathbf{k}$ ) components of  $\Pi^{\rho\mu}$  have a tensor structure proportional to the tensor  $T^{\rho\mu} \equiv (g^{\rho\mu} - h^\rho h^\mu + l^\rho l^\mu)$ , where  $g^{\rho\mu} = \text{diag}(1, -1, -1, -1)$  is the signature tensor. This choice of  $T^{\rho\mu}$  allows us to describe the two remaining directions orthogonal to  $h$  and  $l$ . In the basis, the polarization tensor has the following form

$$\Pi^{\rho\mu}(K) = \pi_l(K) L^{\rho\mu} + \pi_t(K) T^{\rho\mu}, \quad (5)$$

where the longitudinal polarization function is defined as  $\pi_l = (1 - \omega^2/k^2) \Pi^{00}$  and the transverse polarization function is found to be  $\pi_t = (g_{\rho\mu} \Pi^{\rho\mu} - \pi_l)/2$ . In the leading order in the fine structure constant, both longitudinal and transverse polarization functions are real for  $K^2 \equiv \omega^2 - k^2 > 0$ . For an ultrarelativistic degenerate gas of electrons, they are<sup>2</sup> [8]:

$$\pi_l = \frac{1}{D_e^2} \left(1 - \frac{\omega^2}{k^2}\right) \varphi_l \left(\frac{\omega}{kv_F}\right), \quad \pi_t = \frac{3}{2} \omega_{pe}^2 \varphi_t \left(\frac{\omega}{kv_F}\right), \quad (6)$$

where the electron plasma frequency and the Debye screening distance are defined as

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<sup>2</sup> Our Eq. (6) differs from that given in [8] by an extra factor  $(1 - \omega^2/k^2)$  because our definition of  $\pi_l$  differs from that used by Braaten and Segel.

$$\omega_{pe}^2 = \frac{4}{3\pi} e^2 \mu_e^2, \quad D_e = \frac{v_F}{\sqrt{3}\omega_{pe}} \simeq \sqrt{\frac{\pi}{4e^2}} \frac{1}{p_F},$$

with  $\mu_e$  being the chemical potential of electrons and  $p_F \simeq \mu_e$  - the Fermi momentum of electrons;  $v_F \simeq 1$  is the electron Fermi velocity. We introduce also the following notations:

$$\varphi_l(x) = 1 - \frac{x}{2} \ln \frac{x+1}{x-1}$$

is the Lindhard's function; and

$$\varphi_t(x) = 1 + (x^2 - 1) \varphi_l(x).$$

The axial polarization tensor is given by

$$\Pi_5^{\rho\mu}(K) = \pi_A(K) i h_\lambda \epsilon^{\rho\mu\lambda 0},$$

where  $\epsilon^{\rho\mu\lambda 0}$  is the completely antisymmetric tensor ( $\epsilon^{0123} = +1$ ), and the axial polarization function  $\pi_A(K)$  is of the form (see e.g. Ref. [9] and the references therein)

$$\pi_A(K) = \frac{3}{2} \frac{\omega_{pe}^2}{\mu_e} \left( 1 - \frac{\omega^2}{k^2} \right) \varphi_l\left(\frac{\omega}{kv_F}\right)$$

The in-medium photon propagator has the same tensor structure as the polarization tensor. It can be found from Dyson's equation. By the use of the Lorentz gauge we obtain:

$$D_{\lambda\rho}(K) = \frac{4\pi}{K^2 - \pi_l} L_{\lambda\rho} + \frac{4\pi}{K^2 - \pi_t} T_{\lambda\rho}. \quad (7)$$

The total energy of neutrino-pair is  $\omega \sim T \ll \mu_e$ . The four-momentum of a neutrino pair is time-like  $K^2 > 0$ . When  $\omega > k$ , polarization functions differ in the order of magnitudes, namely,  $\pi_A \sim \pi_{l,t} T / \mu_e$ . Therefore, the axial polarization of the medium can be neglected. Insertion of (5) and (7) into (2) with the above approximation yields the effective weak current of an electron in the medium.

$$J^\mu = \bar{\psi} \gamma^\mu (c_V - c_A \gamma_5) \psi + c_V \left( \bar{\psi} \gamma_\lambda \psi \right) \left[ F_l(\omega, k) L^{\lambda\mu} + F_t(\omega, k) T^{\lambda\mu} \right]$$

The contribution of longitudinal virtual photons is proportional to the following factor

$$F_l(\omega, k) \equiv \frac{\pi_l}{\omega^2 - k^2 - \pi_l} = -\frac{\varphi_l(\omega/kv_F)}{D_e^2 k^2 + \varphi_l(\omega/kv_F)}.$$

The contribution of transverse virtual photons is proportional to

$$F_t(\omega, k) \equiv \frac{\pi_t}{\omega^2 - k^2 - \pi_t} = \frac{\frac{3}{2}\omega_{pe}^2 \varphi_t(\omega/kv_F)}{\omega^2 - k^2 - \frac{3}{2}\omega_{pe}^2 \varphi_t(\omega/kv_F)}$$

The poles of this expression give the eigenmodes of oscillations for the electron plasma, which frequency  $\omega(k)$  is larger than the plasma frequency of electrons. At temperatures  $T < T_m \ll \omega_{pe}$  we are considering, the number of such excited oscillations in the medium is exponentially suppressed, therefore contributions from the poles can be neglected. Further, at  $x \sim 1$  one has  $\varphi_l(x) \sim 1$  and  $\varphi_t(x) \sim 1$  while  $D_e^2 k^2 \lesssim D_e^2 T_m^2 \lesssim 10^{-2}$  as well as  $\omega^2 - k^2 \lesssim T^2 \ll \omega_{pe}^2$ . Therefore, with good precision it is possible to approximate the mentioned factors as  $F_l \simeq -1$  and  $F_t \simeq -1$ . We face the remarkable observation, that under conditions of strong degeneration of the medium and small momentum transfer, the collective contribution of electrons to the effective weak current of a testing electron does not depend on parameters of the electron plasma. With this simplification we obtain

$$J^\mu = \bar{\psi}\gamma^\mu(c_V - c_A\gamma_5)\psi - c_V \left(\bar{\psi}\gamma_\lambda\psi\right) \left(L^{\lambda\mu} + T^{\lambda\mu}\right). \quad (8)$$

By taking into account the following identity

$$L^{\lambda\mu} + T^{\lambda\mu} \equiv g^{\lambda\mu} - h^\lambda h^\mu,$$

and conservation of the electromagnetic current of the initial electron

$$K^\lambda \left(\bar{\psi}\gamma_\lambda\psi\right)_{fi} = 0,$$

we find that the effective vector weak current of the electron vanishes. Only the axial-vector contribution survives, and the effective weak current of electron in the medium has the following form

$$J^\mu = -c_A \bar{\psi}\gamma^\mu\gamma_5\psi.$$

Having obtained this result, it is easy to obtain the correct neutrino emissivity caused by electron-phonon interaction in a neutron star crust. The neutrino-pair emissivity previously calculated in [3], [4], [5] can be written in the following form

$$Q_0 = \frac{8\pi G_F^2 Z^2 e^4 C_+^2}{567} T^6 n_i L, \quad (9)$$

where  $L$  is a known slowly varying function of temperature, density, and the crystal structure<sup>3</sup>. Because of a shortage of a place we do not show this function explicitly. The most general, explicit expression of the function  $L$  was given in [10].  $C_+$  is a combination of the weak coupling constants which adds contributions of electron, muon and tau neutrinos. It is defined as  $C_+^2 \equiv c_V^2 + c_A^2 + 2(c_V'^2 + c_A'^2)$ . To take into account the above collective effects, one has only to replace by zero  $c_V$  and  $c_V'$  in the neutrino-pair emissivity (9). Thus, we obtain

$$Q = \frac{8\pi G_F^2 Z^2 e^4 C_{eff}^2}{567} T^6 n_i L. \quad (10)$$

where  $C_{eff}^2 \equiv c_A^2 + 2c_A'^2 = \frac{3}{4}$ . To estimate the efficiency of the collective effects, let's compare our result (10) with the previous formula (9):

$$\frac{Q}{Q_0} = \frac{c_A^2 + 2c_A'^2}{c_V^2 + c_A^2 + 2(c_V'^2 + c_A'^2)} = 0.448. \quad (11)$$

This ratio demonstrates the large importance of collective processes in the crust of a neutron star. Production of neutrinos due to electron-phonon interaction is suppressed by collective effects. The  $\nu\bar{\nu}$  emissivity is less in approximately 2.23 times than previously estimated.

## ACKNOWLEDGMENTS

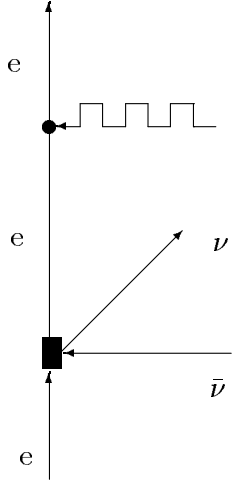
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<sup>3</sup>In the case of a liquid phase, this function amounts to the Coulomb logarithm.

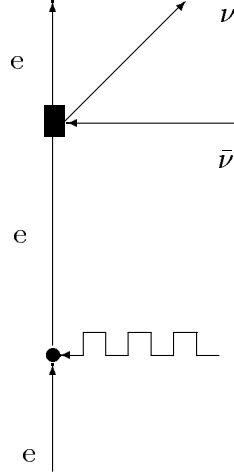


## REFERENCES

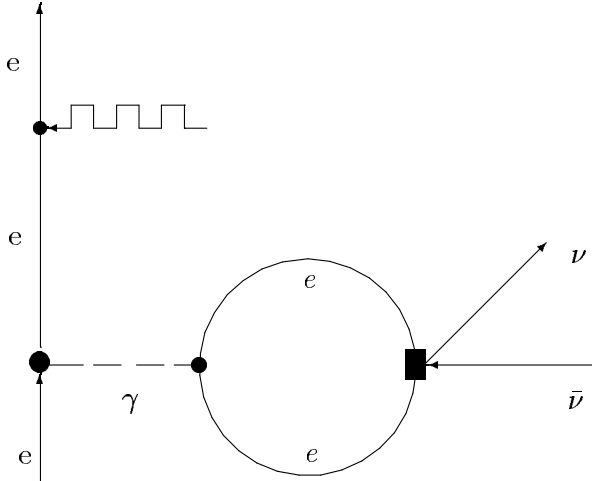
- [1] J.M. Lattimer, K. Van Riper, M. Prakash, ApJ, 425 (1994) 802.
- [2] C. J. Pethick and V. Thorsson, Phys. Rev. Lett., 72 (1994) 1964.
- [3] E. Flowers, ApJ, 180 (1973) 911.
- [4] N. Itoh, Y. Kohyama, N. Matsumoto, M. Seki, ApJ, 285 (1984) 304.
- [5] N. Itoh, H. Hayashi, A. Nishikawa, Y. Kohyama, ApJ Suppl., 102 (1996) 411.
- [6] J. M. Ziman, Electrons and Phonons, Oxford University Press: Oxford 1960.
- [7] H. Nagara, Y. Nagata, and T. Nakamura., Phys. Rev., A36 (1987) 1859.
- [8] E. Braaten and D. Segel, Phys.Rev., D48 (1993) 1478.
- [9] G. G. Raffelt, "Stars as laboratories for fundamental physics: the astrophysics of neutrinos, axions, and other weakly interacting particles", The University of Chicago Press, 1996.
- [10] D. G. Yakovlev and A. D. Kaminker, Astronomy Letters, 22 (1996) 491; Translated from Pis'ma v Astronomicheskii Zhurnal, 22 (1996) 549.



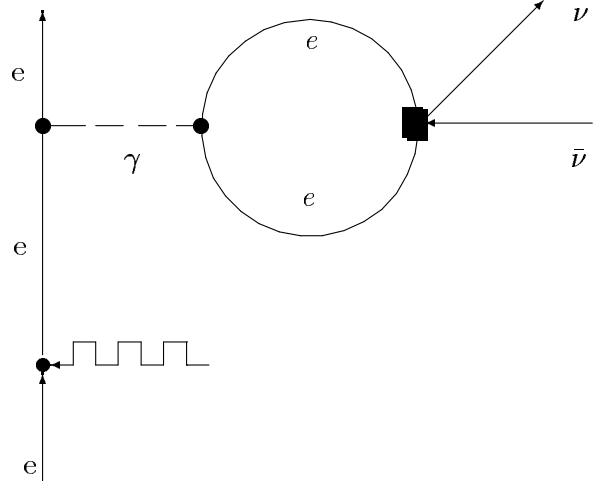
1a



1b



1c



1d

Fig. 1

Fig 1. Lowest order Feynman diagrams contributing to the matrix element of neutrino-pair production. Diagrams 1a and 1b include vacuum weak interaction. The broken line is a phonon. Diagrams 1c and 1d, with the electron loop, describe the weak interaction of the initial electron with the neutrino field via medium polarization. The intermediate virtual

photon is shown as a dashed line.

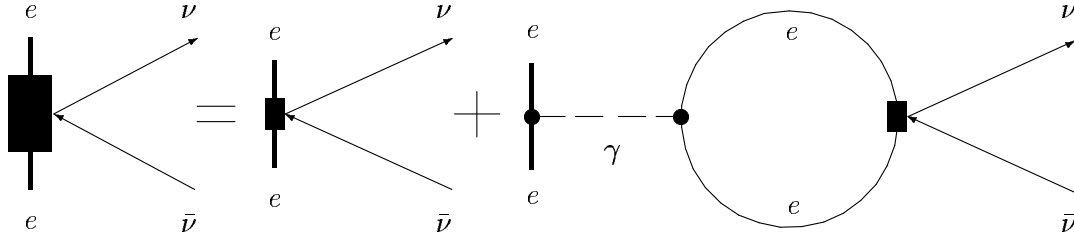


Fig. 2

Fig 2. Feynman graph insertions contributing to the sum of diagrams 1a and 1c and in the total of diagrams 1b and 1d of Fig. 1 as well. The first diagram represents the vacuum weak interaction. The second diagram, with the electron loop, describes interaction via an intermediate photon, shown by a dashed line. The sum of these diagrams is the effective in-medium weak interaction of an electron with a neutrino field.